Svar till valda uppgifter i Johnson & Wichern 1998

4.1 a) $p = 2$, $\boldsymbol{\mu} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$\boldsymbol{\Sigma} = \begin{pmatrix} 2 & -0.8\sqrt{2} \\ -0.8\sqrt{2} & 1 \end{pmatrix}$$

$|\boldsymbol{\Sigma}| = 0.72$

$$\boldsymbol{\Sigma}^{-1} = \begin{pmatrix} \frac{1}{0.72} & \frac{\sqrt{2}}{0.9} \\ \frac{\sqrt{2}}{0.9} & \frac{2}{0.72} \end{pmatrix}$$

$$f(\boldsymbol{x}) = \frac{1}{2\pi\sqrt{0.72}} \exp(-\frac{1}{2}[\frac{(x_1-1)^2}{0.72} + \frac{2\sqrt{2}}{0.9}(x_1-1)(x_2-3) + \frac{2(x_2-3)^2}{0.72}])$$

b)

$$\frac{(x_1-1)^2}{0.72} + \frac{2\sqrt{2}}{0.9}(x_1-1)(x_2-3) + \frac{2(x_2-3)^2}{0.72}$$

4.2 a) $p = 2$, $\boldsymbol{\mu} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\boldsymbol{\Sigma} = \begin{pmatrix} 2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1 \end{pmatrix}$$

$|\boldsymbol{\Sigma}| = 3/2$

$$\boldsymbol{\Sigma}^{-1} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{-\sqrt{2}}{3} \\ \frac{-\sqrt{2}}{3} & \frac{2}{\sqrt{3}} \end{pmatrix}$$

$$f(\boldsymbol{x}) = \frac{1}{2\pi\sqrt{3/2}} \exp(-\frac{1}{2}[\frac{2x_1^2}{3} - \frac{2\sqrt{2}}{3}x_1(x_2-2) + \frac{4(x_2-2)^2}{3}])$$

b)

$$\frac{2x_1^2}{3} - \frac{2\sqrt{2}}{3}x_1(x_2-2) + \frac{4(x_2-2)^2}{3}$$

c) $c^2 = \chi_2^2(0.5) = 1.39$. En ellips centrerad i $(0, 2)'$ med storaxel med halvlängd $\sqrt{\lambda_1}c = \sqrt{2.366}\sqrt{1.39} = 1.81$ i riktning $\boldsymbol{e}_1 = (0.888, 0.460)'$ och lillaxel med halvlängd $\sqrt{\lambda_2}c = \sqrt{0.634}\sqrt{1.39} = 0.94$ i riktning $\boldsymbol{e}_2 = (-0.460, 0.888)'$.

4.3 a) Nej b) Ja c) Ja d)Ja e) Nej

4.4 a) $3X_1 - 2X_2 + X_3 \in \mathcal{N}(13, 9)$. b) Varje $\boldsymbol{a}' = (3 - 2a_3, a_3)$ exempelvis $\boldsymbol{a}' = (1, 1)$.

4.5 a) $X_1|X_2 = x_2 \in \mathcal{N}((x_2-2)/\sqrt{2}, 3/2)$. b) $X_2|X_1 = x_1, X_3 = x_3 \in \mathcal{N}(-2x_1 - 5, 1)$. c) $X_3|X_1 = x_1, X_2 = x_2 \in \mathcal{N}((x_1 + x_2 + 3)/2, 1/2)$.